

Module 9: Hypothesis Testing

The "effect" of a particular factor on some health outcome can be described as a parameter. Statistical hypothesis testing begins with a probability model assuming there is no effect or a **null hypothesis (H_0)** and deciding whether there is sufficient evidence to reject the null hypothesis in favor of the **alternative hypothesis (H_1)**. Think of it like a legal trial; innocent until proven guilty.

One-sided Hypothesis: $H_0: \theta = \theta_0$ vs $H_1: \theta > \theta_0$ (or $\theta < \theta_0$)

Two sided Hypothesis: $H_0: \theta = \theta_0$ vs $H_1: \theta \neq \theta_0$

We test H_0 by finding an appropriate subset $R \subset x$, **the rejection region**. R is defined by $R = \{x : T(x) > c\}$; where T is a **test statistic** and c is a **critical value**.

- If $X \in R \rightarrow$ reject null hypothesis
- If $X \notin R \rightarrow$ retain the null hypothesis

Errors

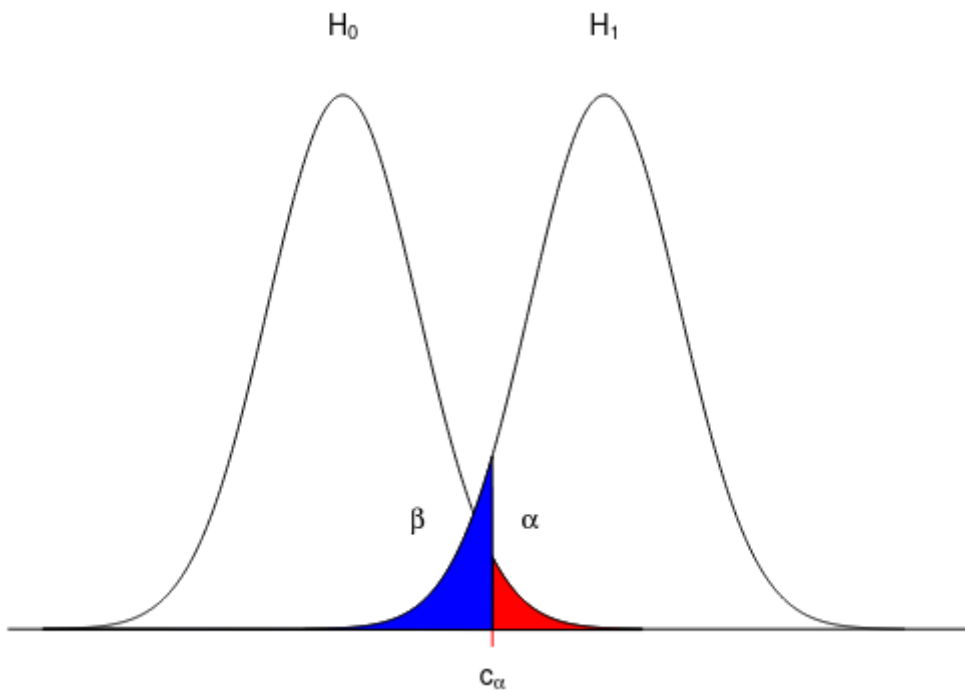
| | | Condition (gold standard) | |
|------------|----------|---------------------------|---------------------|
| | | Absent | Present |
| Index test | Negative | False positive rate | False negative rate |
| | Positive | | |

$$\alpha = 1 - \text{specificity}$$

$$1 - \beta = \text{sensitivity}$$

Type I error or **false positive** is rejecting H_0 when it is true. α represents the probability of this error (typically set at .05)

Type II error or **false negative** is failing to reject the null when it is false. Probability represented by β .



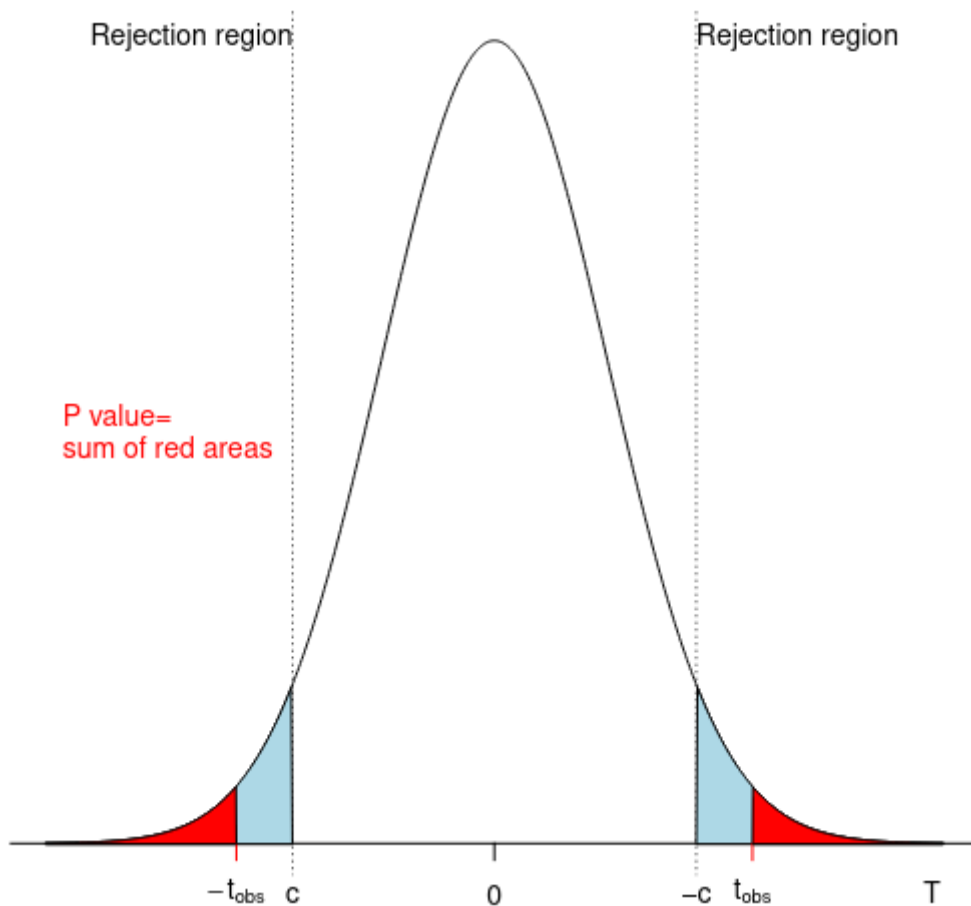
Conducting and Interpreting the Test

1. Define null and alternative hypothesis
2. Set a desired α level. We choose a critical value such that: $P(T(x) > c \mid \theta = \theta_0) = \alpha$
3. Collect data
4. Calculate the observed test statistic value and compare to the critical value
5. Make decision

P-value is the smallest critical value at which the test leads to rejecting the null hypothesis. It is a probability, when assuming the null is true, of obtaining a test statistic at least as large as the one we observed.

- $p < \alpha \rightarrow$ reject null
- $p > \alpha \rightarrow$ retain null

Retaining the hypothesis does not mean the null is true, it is interpreted as a lack of evidence to accept the alternative.



We expect the data to come from the center of the distribution, there is a lower probability of pulling data from tails. If our sample statistic occurs at the tail, then this may not be a representative distribution.

- The **power function** of a test with rejection region R is:
 - $Q(\theta) = P(X \in R)$
- The **size** of a test is defined as:
 - $S(\theta) = \sup(Q(\theta))$
- A test is said to have **level α** if its size is less than or equal α
- $1 - \beta$ is called the **power**, the probability of rejecting the null when the alternative of true
 - The alternative corresponds to a range of value, thus power will depend on whatever alternative parameter value we choose to consider
- These quantities allow identifying the adequate critical value for the test

If variance is known we can use Z-scores to compare to our critical value:

$$z = \frac{\bar{X} - \mu_0}{\sqrt{\sigma^2/n}}$$

Otherwise use a t-distribution with $n-1$ degrees of freedom. A paired t-test is used when we are interested in the difference between two variables for the same subject when the null is that the

mean difference is 0:

$$t = \frac{\bar{d}}{\sqrt{S_d^2/n}}$$

\bar{d} represents the difference in sample mean differences and the denominator is the standard error.

Likewise, we can also use t and z scores to create tests for proportion:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Relationship to Confidence Intervals

The null hypothesis can be tested against a two-sided alternative using a significance level α by assessing if the null parameter is contained in the $100(1 - \alpha)\%$ confidence interval for the parameter.

Also consider the comparison between two population means. If a confidence interval of the differences may not include zero while the CI of each population mean may overlap. So, overlapping intervals do not imply the samples have the same means.

Revision #9

Created 25 August 2022 13:57:14 by Elkip

Updated 25 August 2022 15:44:37 by Elkip