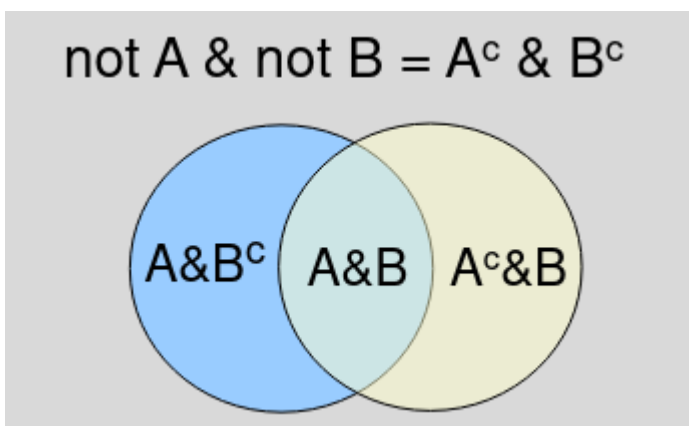


# Module 2: Probability

Probability is the study of uncertainty and randomness in the world. It measures chance.

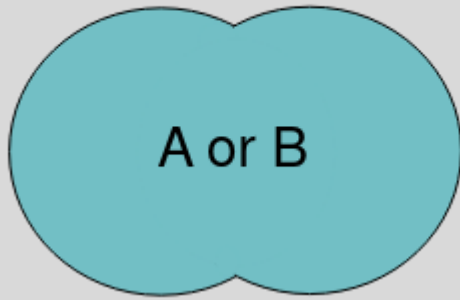
Proportion is a summary statistic. Proportion measures size (i.e. how many patients have optional blood pressure).

- We show the probability of Event A as  $P(A)$
- The complement of an event, or chance of event not occurring is  $A^C$ 
  - $P(A) + P(A^C) = 1$
- Two events are mutually exclusive (or "disjoint") if when one event occurs the other cannot
- Two events are independent if the probability of one has no impact on the occurrence of the other
  - Events are independent if:
    - $P(A|B) = P(A)$
    - $P(A|B) = P(A | \text{not } B)$
    - Odds ratio = 1 (binary outcome)
- If two events are not independent they are said to be statistically associated
- Joint Probability =  $P(A \& B) = P(A, B) = P(A \cap B)$ 
  - If A and B are independent events:  $P(A \& B) = P(A) * P(B)$



- $P(A \text{ or } B) = P(A \cup B)$ 
  - When A and B are non-mutually exclusive  $P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$
  - When A and B are mutually exclusive events  $P(A \text{ or } B) = P(A) + P(B)$  since  $P(A \& B) = 0$

$$\text{not } A \text{ \& not } B = A^c \text{ \& } B^c$$



- Conditional probability is the chance of an event occurring given that another event occurred, written as  $P(A|B)$  = The probability of A given B.
  - $P(A|B) = P(A \& B) / P(B)$
  - $P(A|B) * P(B) = P(A \& B)$

## Bayes' Theorem

Because  $P(A \& B) = P(B \& A)$ ,  
 $P(A \& B) = P(B \& A) = P(B|A) \times P(A)$   
 $=$   
 $P(A|B) \times P(B) = P(B|A) \times P(A)$

$$\mathbf{P(B|A) = P(A|B) * P(B) / P(A)}$$

## Sensitivity

Sensitivity of a screening test = Probability of positive test given the person has the disease. If X is the test result and Y represents if the person actually has the disease, this can be expressed as  $P(X = + | Y = +)$  = the probability of a disease given the test was positive.

$$\text{Sensitivity} = \text{True Positive Fraction} = P(\text{Test}+ | \text{Disease})$$

$$\text{Specificity} = \text{True Negative Fraction} = P(\text{Test} - | \text{No Disease})$$

$$\text{False Positive} = P(\text{Test} + | \text{No Disease})$$

$$\text{False negative} = P(\text{Test} - | \text{Disease})$$

$$\text{Positive Predictive Value} = P(\text{Disease} | \text{Test} +)$$

$$\text{Negative Predictive Value} = P(\text{No Disease} | \text{Test} -)$$

## Odds Ratio

Odds ratio can be used to check independence, events are independent when  $OR=1$ .

$$OR = ( P(X = + | Y = +) / P(X = - | Y = +) ) / ( P(X = + | Y = -) / P(X = - | Y = -) )$$

$$= ( P(X = + | Y = +) * P(X = - | Y = -) ) / ( P(X = + | Y = -) * P(X = - | Y = +) )$$

Symmetry of Odds Ratio

$$OR = \frac{P(D | E)}{P(\text{not } D | E)} \div \frac{P(D | \text{not } E)}{P(\text{not } D | \text{not } E)} = \frac{P(E | D)}{P(\text{not } E | D)} \div \frac{P(E | \text{not } D)}{P(\text{not } E | \text{not } D)}$$

---

Revision #5

Created 16 August 2022 13:51:52 by Elkip

Updated 16 August 2022 15:55:07 by Elkip