Module 13: Linear Regression

Correlation and regression attempt to describe the strength and direction of the association between two (or more) continuous variables.

Pearson Correlation

Recall *r* is an estimate of population correlation coefficient:

$$r = \frac{\sum_{i=1}^{N}(y_{1i} - \overline{y_1})(y_{2i} - \overline{y_2})}{\sqrt{\sum_{i=1}^{N}(y_{1i} - \overline{y_1})^2 \sum_{i=1}^{N}(y_{2i} - \overline{y_2})^2}}$$

It is always between -1 and 1. With 0 indicating no positive or negative linear relationship between the variables.

> A strong correlation does not imply causality.

It indicates the strength and direction of a linear relationship between two random variables. The square of r, r^2 = R, measures how much information is shared between two variables; It is also called the coefficient of determination.

> R^2 can be explained as the proportion of the variability in y that can be explained by the independent variables (X).

r can also be expressed as the average product in standard units in terms of sample standard deviations:

$$r = \frac{1}{n-1}\sum_{i=1}^{N}\left(\frac{y_{1i} - \overline{y_1}}{s_{y1}}\right)\left(\frac{y_{2i} - \overline{y2}}{s_{y2}}\right)$$

Assumptions for Pearson's Correlation:

- Observations are independent

- The association is linear
- Variables are approximately normally distributed

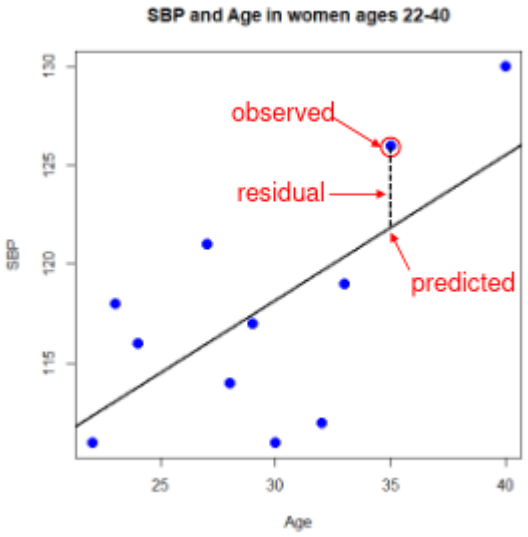
We can compute a test statistic for r with a t-distribution:

t = r / se(r); Where SE of r = sqrt((1-r^2)/(n-2))

Note that se is inversely related to n, so a large sample size results in a smaller se(r). Also the test has n-2 degrees of freedom.

Simple Linear Regression

Linear regression is used to quantify the relationship between one or more independent variables (X) and a single dependent variables (Y). Simple linear regression is the case when we have 1 independent continuous variable and 1 dependent continuous variable.



The line of best fit is the line which minimized the least squares (LS) estimate:

$$\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

The sum of predicted minus observed values squared. For regressions with only one independent variable, X, this yields to the following equation:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

We can take the derivative with respect to each beta and set equal to 0 to end up with the following 2 equations:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} x_i\right)/n}{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2/n} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} = r_{xy}\frac{s_y}{s_x}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Even after we find our best fit line, we cannot predict values that were outside of our sample range.

Estimated Variances of Estimates

$$\widehat{\sigma^2} = \frac{RSS}{n-2} = \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{n-2}$$

$$Var(\hat{\beta}_1 | X) = \frac{\sigma^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$Var(\hat{\beta}_0 | X) = \sigma^2 \left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}\right)$$

The square root of estimated variance is Standard Error (SE).

In R, the lm() function can be used to determine the simple linear regression:

> res <- lm(var_y ~ var_x, data=mydata)

Sum of Squares

Model SS - SS of the differences between y predicted by the model and the overall average. (y_hat - y_bar)^2

Error SS - SS of the differences between y observed and the y predicted by the model. (yi - y_hat)^2

Total SS - SS of the differences between y observed by the model and the overall average. (yi - y_bar)^2

The better the model fits the larger the model SS and the smaller the Error SS.

F Values

$$F = \frac{Model\ SS\ /\ Model\ d.f.}{Error\ SS\ /\ Error\ d.f.}$$

The numerator df is for the model and the denominator df is for error. In a situation with one 1 X variable, the F-test is equivalent to the t-test for the null hypothesis that $\beta_1 = 0$.

Also notice that in the case of one predictor, F is the square of t.

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